

**Final Technical Report**

ARO Grant # DAAD 19-99-1-0242

**Secondary Saddle Points and Vortex Asymmetry in Three Dimensional Flows**

February 28, 2000

By

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**Executive Summary**

In this research perturbations are placed at a variety of spatial locations and their progression in time is tracked together with side force values to link the disturbance of particular regions to the growth of asymmetry. The results of this research provide further evidence that the primary vortex/saddle point region is critical to the formation of vortex asymmetry in conical Navier-Stokes equation set simulations. In addition, the proper technique for tracking the perturbations is explained. Sample perturbation contour plots are provided. The results shown in this report are not complete, but this report gives a gist of what has been achieved as of now. The work continues under support from the University of Cincinnati, exploring perturbation growth rate and propagation in three dimensional flows.

**Research History**

The Principal Investigator initiated the research for this particular grant in July 1999, second year graduate student Dinesh Godavarty assisted. Research funds were expended by December 31, 1999, although the work continues under the support of university resources. This research was a continuation of a previous grant titled "Understanding How Saddle Points Affect the Onset of Vortex Asymmetry".

**Research Goal**

The goal of this research was to determine the flow field region(s) that are the most important to the formation of vortex asymmetry. This was to be accomplished in the framework of three dimensional Navier-Stokes equation set simulations with insight gained from conical Navier-Stokes solutions, as such, it would offer insight into the convective/absolute instability debate. The genesis of this work was previously supported ARO research that indicated flow field saddles play an important role in the formation of vortex asymmetries. Grid resolution studies suggested that both the primary and secondary saddles are critical. It was the role of the current research to explore which region(s) were most critical.

**Research Objectives**

The original objectives of this research were to:

- 1.) observe how field perturbations grow and propagate to form an asymmetric solution from the non-converged symmetric solution,
- 2.) develop growth rate maps,
- 3.) learn whether the primary or secondary saddles are the vortex asymmetry generating regions.

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The initial idea was to apply the conical Navier-Stokes solver used previously to compute a non-converged symmetric solution as a starting point for comparisons. Research specific software would be developed to form difference solutions which would then be visualized. Solutions would be obtained for perturbations placed at many locations in the flow so as to learn the preferred paths for perturbation propagation. In addition, single iteration perturbation solutions would be obtained at a relatively fine grid of solution locations so that perturbation growth rate maps could be generated. From this information the critical regions of the flow field could be ascertained. However, a non-converged symmetric state proved to be somewhat unsatisfactory in the tracking of the perturbation. After these simulations, a converged symmetric state was used as a starting point for the perturbation studies.

### **Analysis Methods**

This section deals with the various methods developed to attempt the visualization of the perturbation propagation in the flow. Each of the methods is explained and the explanation of the usage of the method is also put forward. The physical meaning of the methods is explained.

*Starting point* - The start point for studying the formation of asymmetry is a symmetric solution. A converged asymmetric state can be obtained by one of the following three methods. First the solver can be run for a long time so that the symmetric flow state is perturbed by the round off errors and a machine converged zero asymmetric state is obtained. Second, take a non-converged symmetric solution, physically put in a perturbation until an asymmetric state is obtained. Thirdly, taking a converged symmetric state and then putting in the perturbation to obtain asymmetry. A converged symmetric state is obtained by enforcing symmetric conditions after each iteration until the solver converges to machine zero. The idea behind using a converged state is to eliminate the possibility of the asymmetry growth (albeit very small) interacting with the perturbation and creating noise hence making it difficult to identify the growth of the perturbation.

*Side Force vs. Iteration Number* - The magnitude of the side force vs. Iteration number gives a global picture of the developing asymmetry. This captures small changes in the flow field which might not be visible as such. The comparison of the plots of side force with number of iterations for each case of perturbation location is expected to give important insight into the behavior of the flow field. The side force is calculated by numerically integrating the pressure over the surface of the cone.

*Difference Plots* - A good way to visualize the propagation of the perturbation is to create a plot which is just the difference of two flow field solution files. Difference plots created for this research are not differences from the symmetric initial condition, rather they are differences between the perturbed solution and an unperturbed but developing solution; both have been restarted from the converged symmetric solution. The perturbation in the previous statement means to physically put in a perturbation in the flow field. It was attempted earlier to plot the difference between the "base" solution and the developing perturbation solution. What this means is that although we are not physically perturbing the "base" solution, the "base" solution is perturbed due to the round off error (as mentioned above) and tends to become asymmetric. The difficulty with this methodology is that the entire solution continues to develop, i.e. the perturbed flow solution basically consists of two perturbations, one due to the physical and the other due to the round off errors. The simultaneous development of the latter perturbation leads to "noise" in the flow field. This can be counteracted in part by using a very large perturbation, but this was felt to be undesirable because it might cloud the important physical features being studied. Since this "noise" exists in both; the perturbed as well as the developing solution, it was felt that this can be cancelled out by the difference between the two solutions. Comparisons with the developing unperturbed solution show better perturbation history, at least until the solution becomes very asymmetric and hence different from the developing "base" solution. A third approach was also attempted to visualize the perturbation in which the differences between consecutive solutions were plotted. While this shows the growth of the perturbation it is once again combined with the developing solution changes and hence also undesirable.

*Perturbation Path Maps* - The path traced out by the perturbation as it progresses in time and space is also generated. This is done by taking the average of the perturbation values as the solution progresses in time and space. These plots can be thought of as a time exposed photograph of the stars to get the star trails. The

plots show the path the perturbation takes as it progresses with time. These kinds of plots are also expected to give an insight into the "preferred" path of the asymmetry.

All the above methods can be linked together to analyze the results obtained.

### Results Obtained

Objective 1, namely tracking of the perturbation was satisfied by computing simulations with perturbations placed at many locations in the field. The perturbations were placed at different points in the flow field.

The magnitude of the side force vs. the number of iteration is shown in Figure 1. The behavior of this plot is explained with the help of the density difference plots for the perturbation locations of  $25^\circ$  and  $135^\circ$  as shown in figures 2 and 3. The density time history for the  $135^\circ$  is shown in figure 4. This helps us to visualize and link the flow asymmetry to the difference plots. Figures 5 and 6 are perturbation path maps. Figure 7 shows the path as was deduced from the last two figures. Note that the conclusions drawn from these results are preliminary as the results are not final.

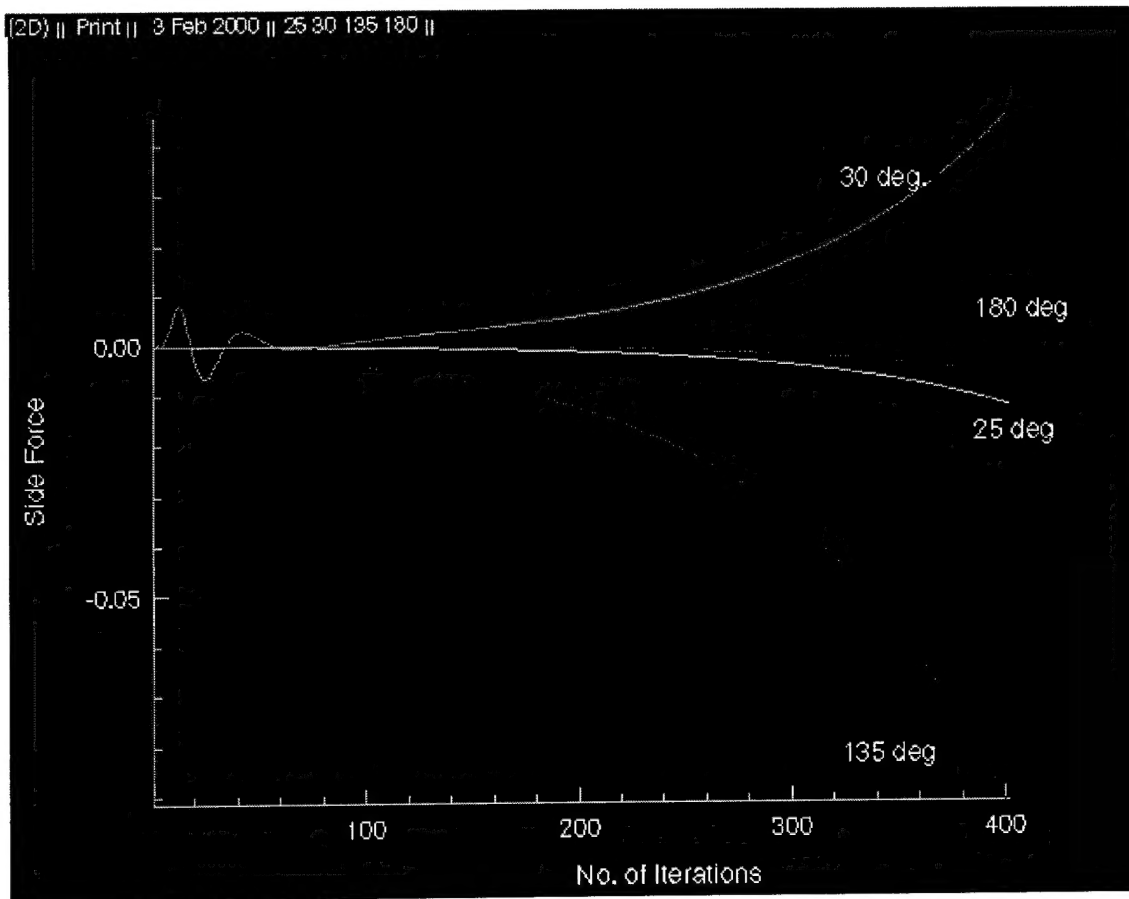


Figure 1. The plot of side force vs. No. of Iterations for different perturbation locations.

The above figure indicates the side force vs. the number of iterations for different cases. The numbers with the lines denote the angle of the perturbation location. Note that the distance of the perturbation location from the cone is not the same for all cases. The numbers on the x-axis indicate the number of iterations from the converged symmetric state i.e. the total number of iterations for this flow state would be the sum of the iterations to get the converged symmetric state and the number of iterations shown on the plot. In this plot, the two most interesting things are the "wiggles" seen in the case of  $30^\circ$  and  $25^\circ$  and the changes in



the direction of the side force for  $25^\circ$ ,  $180^\circ$  and  $135^\circ$  vs. the  $30^\circ$  case. The wiggle can be easily explained with the help of Figures 2 and 3. The initial frames show that the surface of the cone which is closer to the perturbation location feels the perturbation before the other side. The perturbation hits this surface and continues to away from the perturbation location. One part of the perturbation starts to wrap around the cone, all the time exerting some force which is manifested in the form of side force. As this wave continues to propagate, we get the wiggle seen in the side force plot. This wiggle is just the affects of the perturbation propagation. The steady increase in asymmetry is due to vortices becoming asymmetric. The amplitude of the wiggles increase as we increase the perturbation angle. However, the wiggles are absent if we cross  $90^\circ$  because the cone is at an angle of attack and the perturbation is blown away before it actually hits the surface. The amplitude of the wiggle is expected to be a maximum for an angle, which is slightly, less than  $90^\circ$  which was found to be the case. Although the side force plots show that the side force is in the same direction for  $25^\circ$  and  $180^\circ$ , this is not a very significant asymmetry for the number of iterations performed here viz. 400 as compared to  $30^\circ$  and  $135^\circ$ . The switch in the direction for  $30^\circ$  and  $135^\circ$  is quite significant. The critical angle at which such a significant switch occurs is yet to be determined.

Figure 2 and Figure 3 show the difference plots where a perturbation was placed at  $135^\circ$  and  $25^\circ$  from the vertical (clockwise) respectively. The distances of the perturbation locations from the cone are not equal as a large number of combinations of perturbation locations are needed to "zoom" onto the critical location.

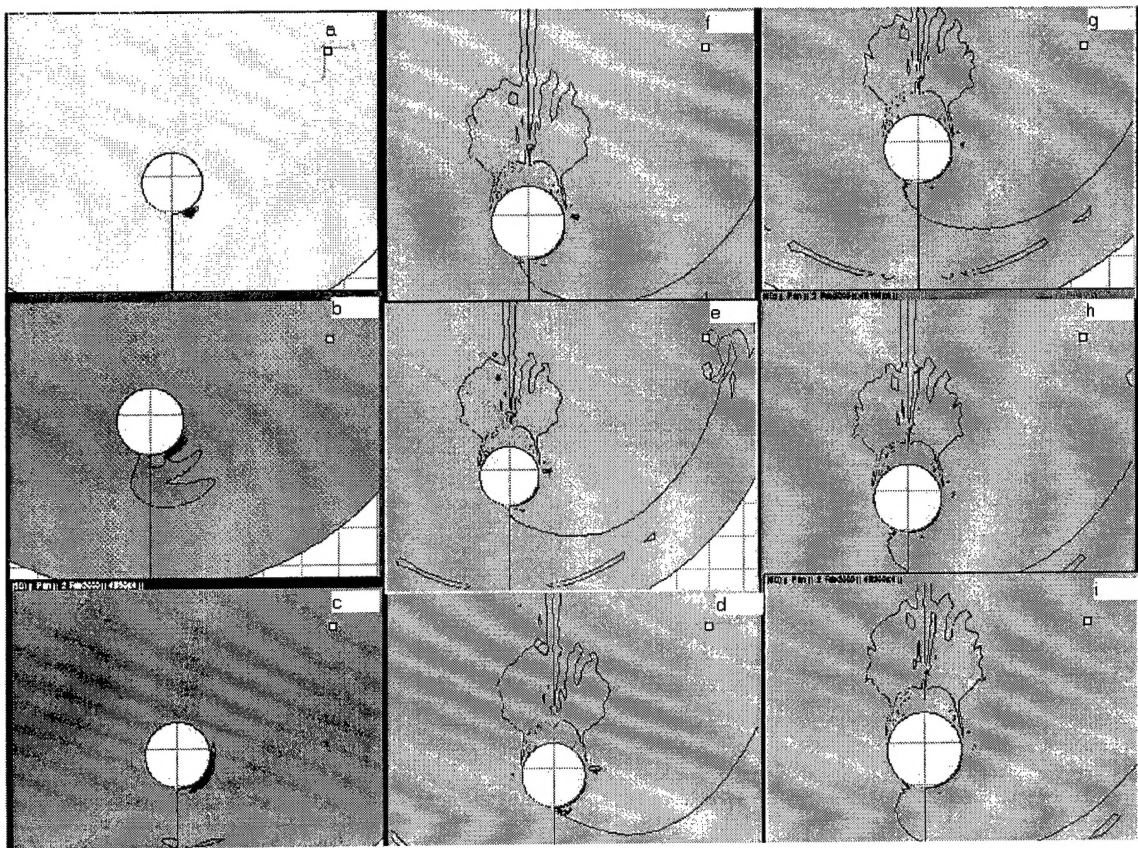
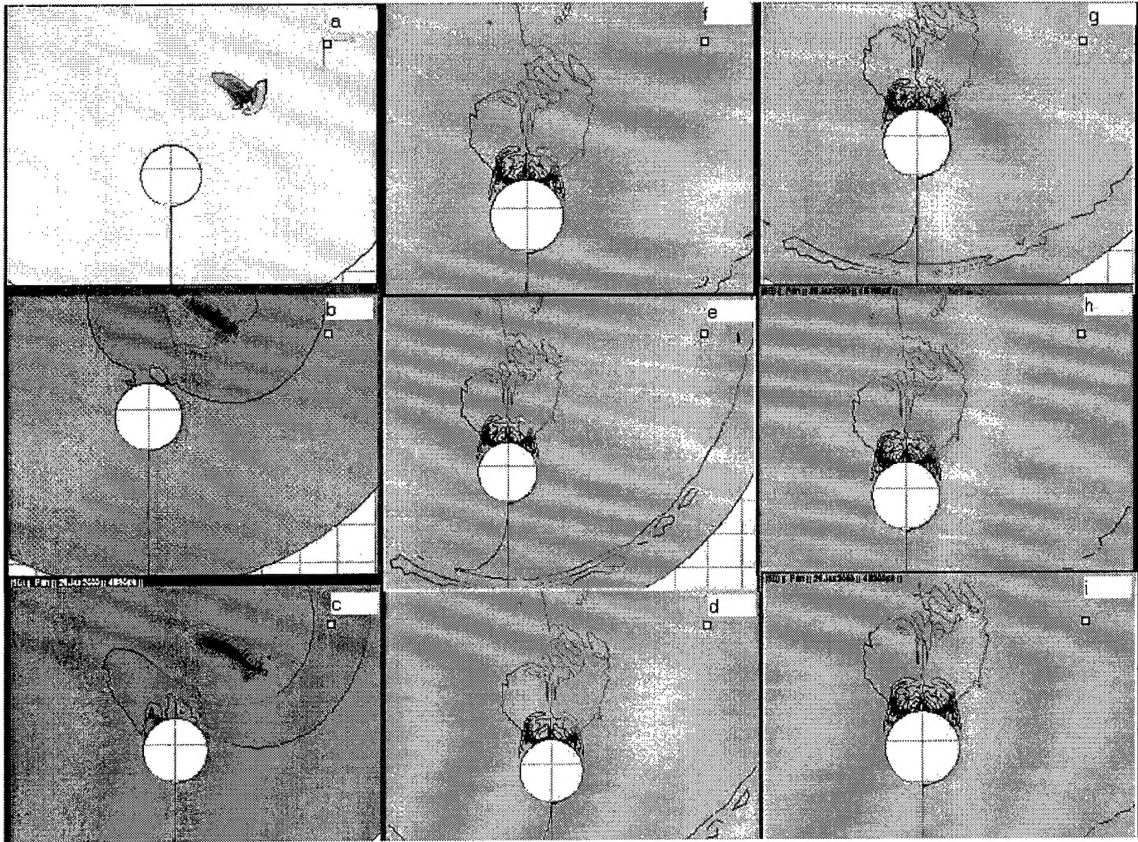


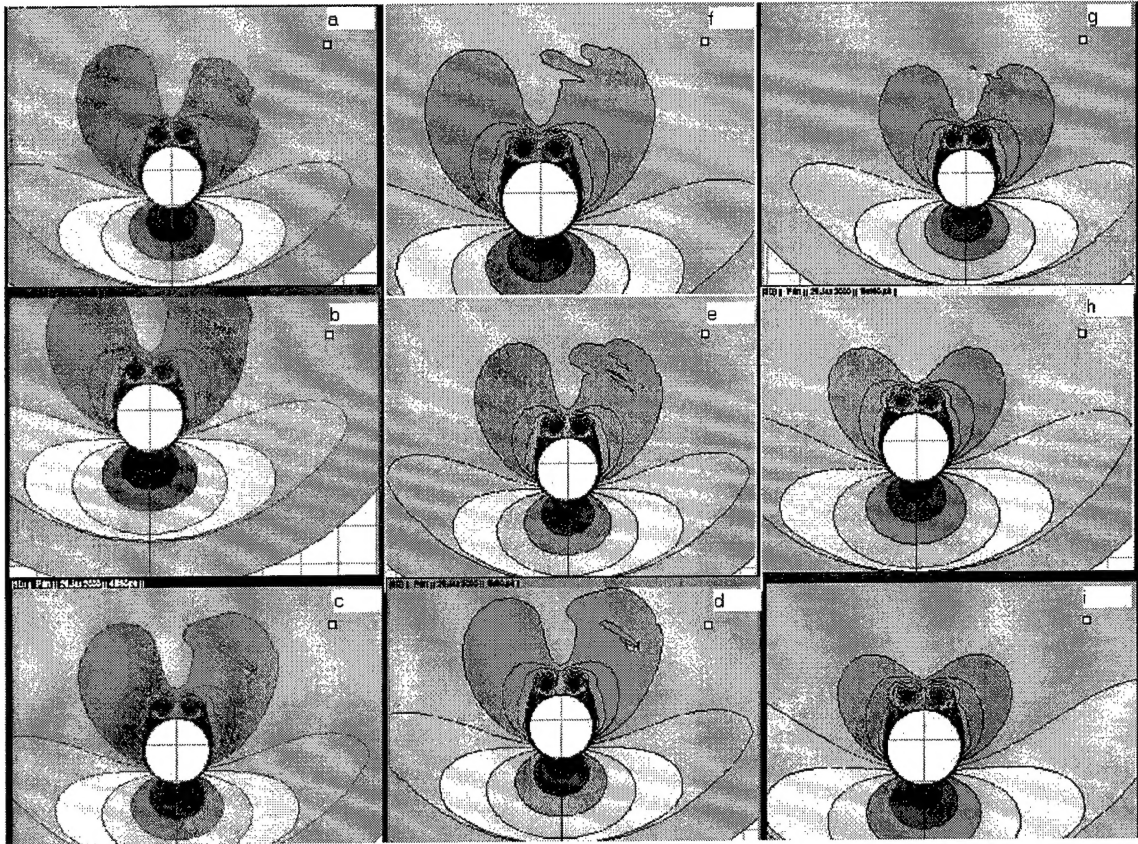
Figure 2 – Contours of Density Difference Comparing Perturbed and Base Solutions for  $135^\circ$ . Solution advances downward in the far left panels, upward in the center panels and downward in the right panels.



*Figure 3 – Contours of Density Difference Comparing Perturbed and Base Solutions for  $25^\circ$ . Solution advances downward in the far left panels, upward in the center panels and downward in the right panels.*

The difference plots seem to show some kind of a source-sink behavior between the two primary vortices. Once the perturbation has been put in, as the solution progresses, we start seeing the asymmetry. For each successive iteration, some kind of a wave seems to emanate from one vortex, loops around and goes into the second vortex. This activity intensifies as the solution progresses. The vortex asymmetry seems to be growing from one vortex to the other. The direction of the side force can be predicted by just identifying the source and the sink in the difference plots. This prediction might prove important in the future.

The changes in the density solution for the  $135^\circ$  case are shown in Figure 4. The perturbation can be seen by the "wiggles" in the cross flow plane streamlines to the right of the cone. Although the surface feels the asymmetry the moment the perturbation hits it, the flow field asymmetry is not apparent until after about one hundred iterations.



*Figure 4 – Contours of Density with the perturbation put in the symmetric Flow State. Solution advances downward in the far left panels, upward in the center panels and downward in the right panels.*

Growth rates were computed at only a few points in the field, as such maps for the entire flow field have not yet been developed. As the perturbation angle is decreased from the vertical, there is a significant switch in the direction of the side force vector for a given number of iterations. The critical angle at which such a switchover occurs is yet to be determined. This critical angle might give the flow location, which "turns" the flow and hence clues to the critical regions of the flow. However, the maps are being created in the continuing research effort for a series of perturbation sizes so as to assess whether the growth rate is independent of perturbation magnitude. This is important for control issues because it is related to control effectiveness and can be useful to control surface designers.

The time averaged values of the density perturbation for the above two perturbation locations have been plotted as shown in Figure 5. The initial perturbation location for the two cases is indicated by the arrows.



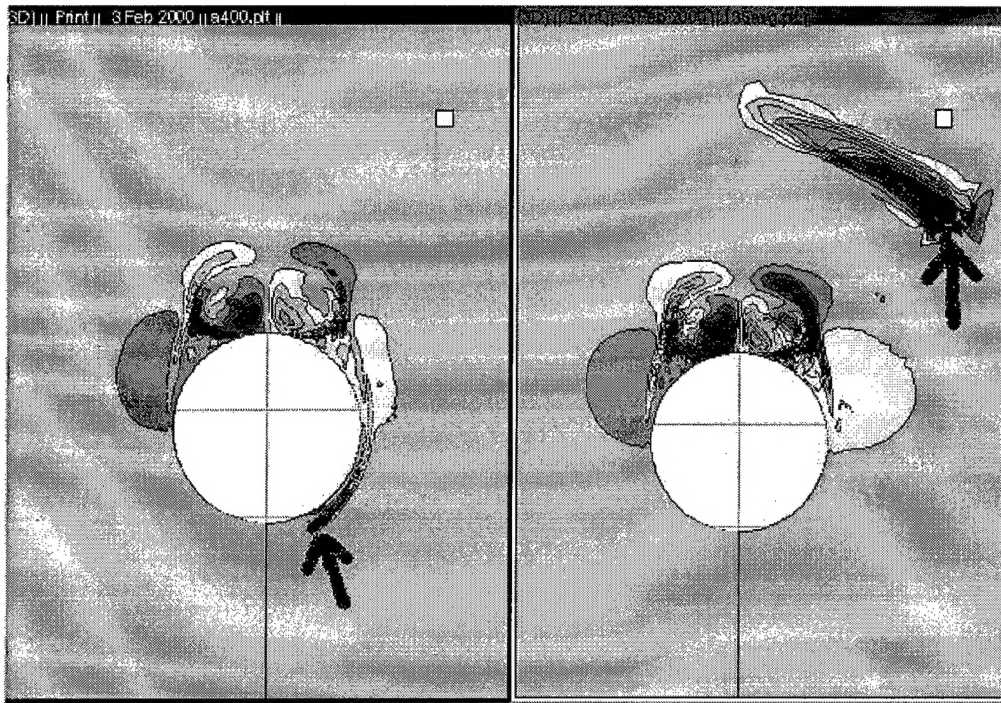


Figure 5 - Time averaged perturbation plots for location  $25^\circ$  and  $135^\circ$ .

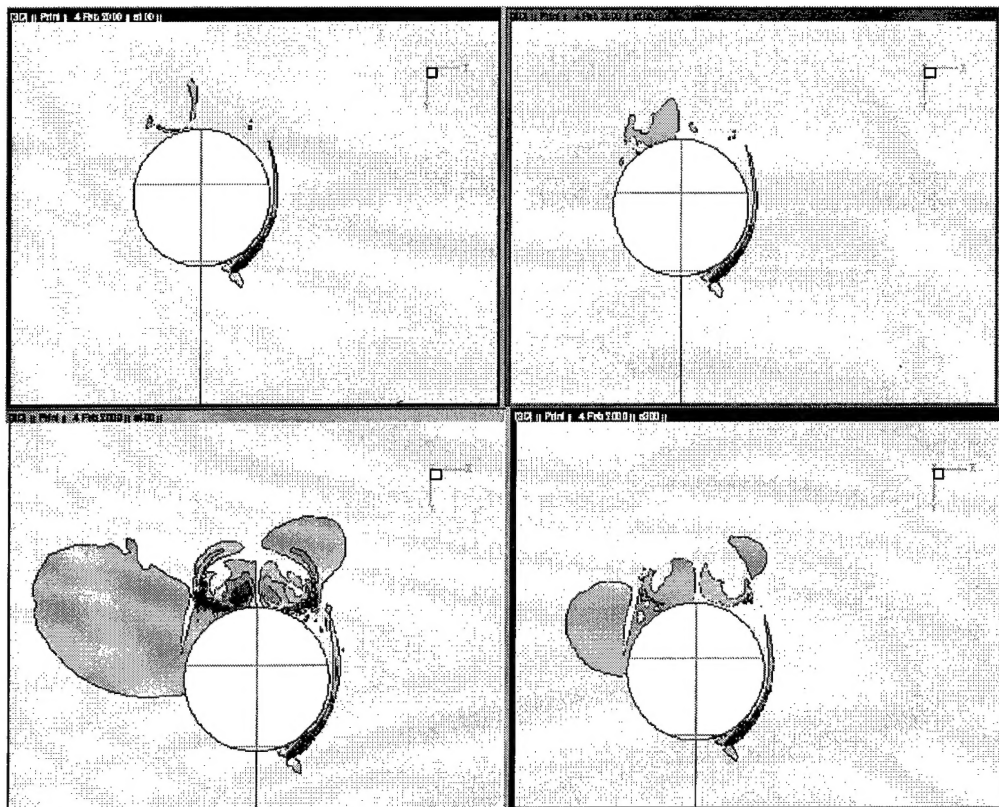
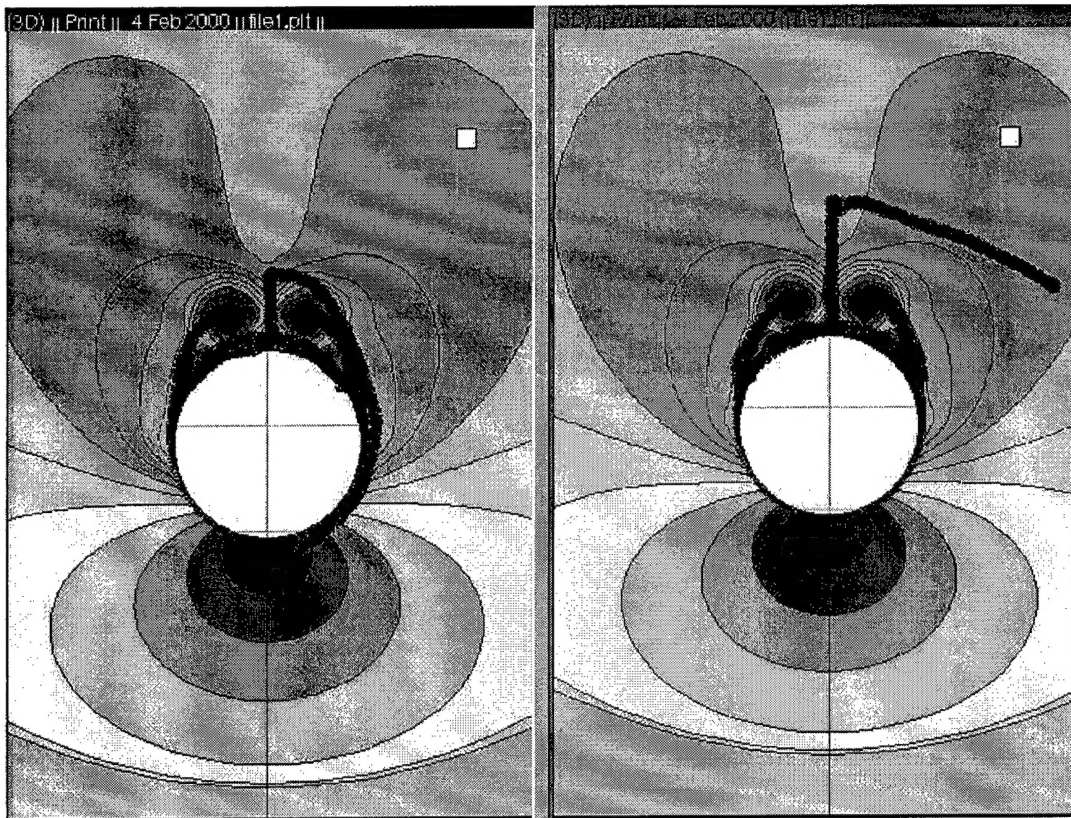


Figure 6 - The average values of the difference solution at 100, 200, 300 and 400 iterations. The figures are to be viewed clockwise.

The time averaged plots for the perturbation location of  $25^\circ$  is shown in Figure 6. This figure indicates how the perturbation progresses with time. The plots are averaged for 100, 200, 300 and 400 iterations. This means that the perturbation plots are averaged for 100, 200 etc. after the perturbation has been put in.

The path followed by the perturbation for the above two cases ( $25^\circ$  and  $135^\circ$ ) is shown by the black line in Figure 7. The two figures indicate that the perturbation first seems to travel through the vortex feeding sheet, interacts with the primary saddle point region goes down towards the surface of the cone, then loops back towards the primary saddle point region amplifying considerably. Although in the case of  $25^\circ$ , the perturbation does travel through the secondary saddle point region, it is only after reaching the primary saddle point region that it's affects apparently become noticeable.



*Figure 7 - The path followed by the perturbation for  $25^\circ$  and  $135^\circ$*

The results obtained to date offer further evidence that the primary saddle region is more important than the secondary, however, additional field perturbation solutions are needed to completely resolve the issue.

#### **Continuing Research**

Research support for activities through June 2000 has been secured from the University of Cincinnati. The following items will be explored:

- 1.) additional perturbation locations and time history plots,

- 2.) detailed growth rate maps at selected field points, developed in part from the results of task 1,
- 3.) more detailed perturbation histories, "zoomed-in" on the important flow features, relating the actual solution, the perturbation solution,
- 4.) several cases at symmetric vortex incidence ratios.
- 5.) time averaged perturbation plots for different perturbation locations
- 6.) 3D simulations for the same flow field

#### **Recommendations for Future Work**

The current work appears to be a rather cost effective way to study asymmetry related stability problems. In this way the issue of convective and absolute instability might be explored.

#### **References**

- 1) *"Understanding How Saddle Points Affect the Onset of Vortex Asymmetry"*, Final Technical Report, ARO Grant # DAAG55-98-1-0328, 1999.
- 2) *"Flow Field Saddles and Their Relation to Vortex Asymmetry"*, Computer and Fluids, Vol. 26, No.5, pp. 505-524, 1997.
- 3) *"On Computing Vortex Asymmetries about Cones at Angle of Attack Using the Conical Navier-Stokes Equations"*, AIAA-93-3268-CP, 1993.